# Fourier analysis

**Requirements**

1. The report should be written in English.
2. Include your student number in each figure title as ‘No. XXXXXXX’. And include your codes in the appendix with the question numbers.
3. Please submit your report in PDF format.

## Fourier transform properties(CTFT)

Create a gate function *g*(*t*), where *D* = 8, *H* = 5 over *t* = [-*D*, *D*] with a time interval 0.001.



* 1. Create a Continuous-time Fourier transform (CTFT) function ***Xw* = CTFT (*x*, *t*, *w*)**, where input the time-domain signal *x* and *t*, and the frequency vector in radians, and output the CTFT coefficients. (Note that here *w* is to represent *ω* in programming.)
  2.  is defined as a time shift of *g*(*t*): . Compare *g*(*t*) and *g*2(*t*) in one plot.
  3. Calculate CTFT of *g*(*t*) and *g*2(*t*), denoted as *Gw* and *Gw*2, using the CTFT function. Compare the module and phase plots, and the real and imaginary plots of *Gw* and *Gw*2 in *w* = -10π~10π, respectively. Verify the time-shift properties of Fourier transform.
  4.  is defined as. Compare *g*(*t*) and over *t* = [-15, 15].
  5. Compare the modulus and phase of CTFT of  and *g*(*t*) in *w*= -10π~10π. Verify the modulation properties of Fourier transform.
  6. According to the Parseval’s formula, calculate and compare the energy of  in both the time and frequency domains. Are they the same? Why?

## DTFT

A gate function *g*(*t*), where *D* = 8, *H* = 2 over *t* = [-*D*, *D*].



* 1. Create a Discrete-time Fourier transform (DTFT) function ***Xw* = DTFT (*nT*, x*n*, *ω*)**, where input the time-domain signal *nT* and *xn* and the frequency vector in radians ***ω***, and output the DTFT coefficients.
  2. Set the sampling interval *T* as *D*/80 and *D*/40, respectively, and sample *g*(*t*) as *g*1[*n*] and *g*2[*n*]. Calculate DTFT of *g*1[*n*] and *g*2[*n*], denoted as *Gw*1and *Gw*2, using the DTFT function. Compare the module and phase of *Gw*1and *Gw*2using different digital frequencies, i.e., *f*, *f*/*f*s, ω/*f* s, in a Nyquist interval.
  3. Deduce the theoretical CTFT function of g(t) and compare it with the *Gw*1and *Gw*2 in *f* = -3*f*s~3*f*s, analyzing the result.
  4. Inverse the DTFT on *Gw*1and *Gw*2 and compare the result with *g*1[*n*] and *g*2[*n*].
  5. Can the Parseval’s relation still be validated, why?

## Windowing effects of DTFT

Create *L*-point *x*(*nT*)*,* (*n* = 0 ~ *L*-1) in Eq. (3), with *T* = 0.01 and *fs* = 100, *f*1 = 16, *A*1 = 1.4, Δ*f* = 1, *f*2 = *f*1 + Δ*f*, *A*2 = 0.6.

*x*(*t*) = *A*1sin(2π*f*1*nT*) + *A*2sin(2π*f*2*nT*) (3)

1. Set *L* = 50, 200 and 1000, respectively. Adopt proper factors to scale magnitudes of DTFT of *x* for each *L*, and indicate *A*1, *A*2 and *f*1, *f*2 in the corresponding magnitude plot in half Nyquist interval [0, *fs*/2]. Fill results in Table 1.

Table 1

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *L* | factor | *A*1 | *A*2 | *f*1 | *f*2 |
| 50 |  |  |  |  |  |
| 200 |  |  |  |  |  |
| 1000 |  |  |  |  |  |

1. Apply Hamming window to obtain windowed signal *x*w, where *L* = 50, 200 and 1000, respectively.Adopt proper factors to scale magnitudes of DTFT of *x*w for each *L*, and indicate *A*1, *A*2 and *f*1, *f*2 in the corresponding magnitude plot in half Nyquist interval [0, *fs*/2]. Fill results in Table 2.

Table 2

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *L* | factor | *A*1 | *A*2 | *f*1 | *f*2 |
| 50 |  |  |  |  |  |
| 200 |  |  |  |  |  |
| 1000 |  |  |  |  |  |

**Hamming window function:**

, where *a*0 =0.53836.

## DFT and FFT

An *L*-point signal, *L* = 10, *y*[*n*] = [-1, 2, 3, 0, -2, 1, 4, -3, 0, -2], *n* = 0 to 9.

* 1. Plot *y*[*n*]
  2. Calculate and show the modulus and phase of DTFT Y(*e*j*ω*) in Nyquist interval [-π, π].
  3. Calculate *N*-point DFT Y(j*ωk*), *N* = 10, and compare with DTFT in (b).
  4. Calculate inverse DFT of Y(j*ωk*) and compare with *y*[*n*].
  5. Zero-padding *y*[*n*] to *L* = 16 and calculate 16-point DFT of *y*[*n*] using FFT, and verify the results by comparing with DTFT in (b).
  6. In order to investigate the computational time of DFT and FFT with respect to *L* of *y*[*n*], (zero-padding *y*), and evaluate the computational time with *L* = [1000:1000:10000]. Show the curve of computational time with respect to *L*.